Indian Statistical Institute, Bangalore Centre
B.Math. (II Year) : 2009-2010
Semester II : Backpaper Examination
Optimization

Time: 3 hours. Maximum Marks : 100

1. [$5 \times 8 = 40$ marks] Prove or disprove:

(i) Feasible set of an LPS is convex.

(ii) Consider an LPS with $(m \times n)$ constraint matrix A. Let $\text{rank}(A) = m \leq n$. Then the feasible set of the LPS can not have more than $\frac{n!}{m!(n-m)!}$ extreme points.

(iii) If the primal problem is unbounded below, then the dual problem is infeasible.

(iv) If the feasible set of an LPS has exactly two extreme points, then it has an optimal feasible solution.

(v) If an LPS has unique optimal feasible solution then the feasible set is compact.

2. [20 marks] Using the simplex method solve: *Minimize* $(-3x_1 + x_2 + 2x_3)$ subject to $3x_1 + x_3 = 6, \quad 2x_1 + x_2 + x_3 = 8, \quad x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$.

3. [15 + 10 marks] Consider the constrained optimization problem: *Maximize* $(x_1 + \cdots + x_n)$ subject to the single constraint $(x_1^2 + \cdots + x_n^2) = 1$.

(i) Justify the use of Lagrangian method for solving this problem.

(ii) Solve the problem using the Lagrangian method.

4. [15 marks] Consider the problem: *Maximize* $(-(x^2 + y^2))$ in $\mathbb{R}^2$ subject to $(x-1)^3 \geq y^2$. Using this example show that the "rank condition" in Kuhn-Tucker theorem can not be dropped.